Harbor porpoise clicks do not have conditionally minimum time bandwidth product

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Abstract: The hypothesis that odontocete clicks have minimal time frequency product given their delay and center frequency values is tested by using an in-phase averaged porpoise click compared with a pure tone weighted with the same envelope. These signals have the same delay and the same center frequency values but the time bandwidth product of the artificial click is only 0.76 of the original. Therefore signals with the same parameters exist that have a lower time bandwidth product. The observation that porpoise clicks are in fact minimum phase is confirmed for porpoise clicks and this property is argued to be incompatible with optimal reception, if auditory filters are also minimum phase.

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1. Introduction

Odontocete whales use echolocation for prey finding and navigation. The transmitter part of the biological sonar system of porpoises and dolphins emits a short pulse, a few tens to about hundred μs long (Au, 1993; Madsen *et al.*, 2004; Madsen *et al.*, 2005; Villadsgaard *et al.*, 2007). Wiersma (1988) notes that these animals' signals, given their very short durations, have a low bandwidth. This does not necessarily mean that they are all narrow band signals as such, but that there is a close coupling between the duration and bandwidth, so that the narrow band high frequency (NBHF) (Morisaka and Connor, 2007) signals, such as those employed by members of the porpoise family, have longer durations than the very broadband signals used by dolphins like Tursiops (Au, 1993). A low time-bandwidth product (TBP) is an advantage in signal detection, because a simple receiver can process it optimally, i.e., function as a matched filter, equivalent to a cross-correlation operation. A signal with low TBP occupies a small rectangular area in a time frequency representation. A filter with an impulse response that is contained within the same area might constitute a matched filter for that signal. The signals with the absolute minimum TBP value of 0.08 belong to a class of signals termed Gabors (Gabor, 1946; Venkatesh *et al.*, 2005) and consist of a Gaussian multiplied by a sinusoid. The notion that odontocete clicks might have a conditionally minimum TBP may have arisen because Gabors start in principle at time minus infinity and are therefore not practically realizable.

Wiersma (1988) used an iterative method to search in artificially generated signals for the one with the lowest TBP. The search was limited to signals with the same delay (see below) and frequency centroid values as real odontocete clicks. He found that the artificial signals that fit these criteria looked like the odontocete clicks from which the delay and frequency centroid values had been obtained. He therefore concluded that these animals have optimum waveforms under these constraints. Here, the TBP of a signal, given these constraining parameters of delay and frequency centroid, is termed the conditionally minimum TBP.

One problem with the analysis carried out by Wiersma (1988) is that the delay value is hard to determine in any noise at all. The frequency centroid is determined as the value that divides the linear amplitude spectrum in two equal halves. In that case the origin is simply zero

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Hz. The delay is analogously determined as the time delay value that divides the signal envelope in two equal halves, but where is time zero? For a recorded sound this is never obvious. Considering that this value is critical to the analysis, extrapolation seems imprudent. One may also wonder if the use of an optimum outgoing signal is important, since the returning echoes are influenced by the objects that reflected them and thus no longer optimal.

The reasoning given in the following describes a test of Wiersma's assertion of conditionally minimum TBP by investigating another artificial signal whose values of delay and frequency centroid are identical to the signal under scrutiny. If the artificial signal has a lower TBP than the real one, then the conditional minimum was not met in the real signal, which then serve as a falsification of the Wiersma's (1988) assertion.

Harbor porpoise signals were chosen because the spectrum of the signal envelope stays well below the frequency centroid. It is not completely impossible to carry out a somewhat similar analysis when this is not the case as were found for most other odontocete species tested, but it complicates things somewhat. It is also crucial to have a very high signal to noise ratio, since otherwise the effective envelope duration gets too high. Porpoises produce clicks that are so similar from one click to the next that it is possible to perform in-phase averaging over a number of consecutively emitted clicks to reduce the noise without noticeably affecting the waveform.

Below the results of this test are explored and it is argued that a minimum TBP signal is not necessarily optimal given the hearing system of the animals.

2. Analysis

We seek a signal with the same delay [*sensu* Wiersma (1988)] and frequency centroid as the porpoise click, $s(t)$. It is possible to describe the porpoise click signal as

$$s(t) = a(t)\cos(\psi(t)),$$

where $a(t)$ is the envelope and $\psi(t)$ is the instantaneous phase function. Regardless of the shape of $\psi(t)$, $a(t)$ will per definition have the same delay value as $s(t)$.

We can therefore construct a new signal, $\hat{s}(t)$, with the same frequency centroid as $s(t)$ by setting $\psi(t)=2\pi f_c t$, where f_c is the frequency centroid of $s(t)$ but keeping $a(t)$. Note that the spectrum of $\hat{s}(t)$ can be found as the convolution between the spectrum of the envelope $a(t)$ and a frequency shift operator, $\delta(f-f_c)$ moving the center of gravity of the envelope spectrum up to the frequency centroid, f_c, of $s(t)$. The amplitude spectrum of the envelope is symmetrical around zero, so if f_c is higher than the highest frequency in the envelope, then $\hat{s}(t)$ has the same frequency centroid as $s(t)$. If not, the spectrum of $\hat{s}(t)$ was not completely moved up to frequencies above the DC border and the resulting frequency centroid is then not necessarily equal to f_c, and the argumentation used here is then not completely valid (see below).

It is not given that $\hat{s}(t)$ has the minimum attainable TBP under the constraining parameters, but if it has a value that is lower than the TBP of $s(t)$ then the assertion that odontocetes use conditionally TBP signals is falsified, since the constructed signal $\hat{s}(t)$ has both the same delay and the same frequency centroid, which were the measured constraints under which TBP should be minimized.

3. Methods

The analysis was carried out with harbor porpoise (*Phocoena phocoena*) clicks recorded at the Fjord & Belt Center in Kerteminde, Denmark. Signals were sampled at 800 kHz in a star shaped array of four hydrophones (Schotten *et al.*, 2004) during a prey finding session with dead fish. The clicks from a sequence of 51 consecutively emitted clicks (Fig. 1) with similar amplitudes (contained within a range below 3 dB, STD=0.8 dB) were averaged in phase (Fig. 2). The selected signals were more powerful at the center hydrophone so they were assumed to have been recorded close to on axis (Rasmussen *et al.*, 2004).

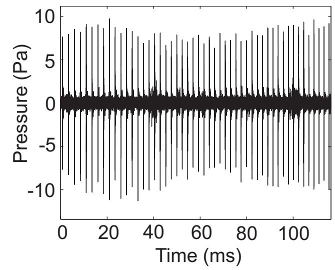

Fig. 1. The click sequence used in the analysis. This is part of a buzz sequence. The rate of click production is approx. 400 Hz.

In performing the averaging, the signals were aligned based simply on the position of the peak pressure value after interpolation to an effective sample rate of 6.4 MHz to improve the quality of the time alignment. The averaged signal was windowed as shown in Fig. 2(a).

4. Results

It is seen from Fig. 2(a) that the aligned signals are remarkably similar. This is confirmed by the relatively small variation in the shape of the amplitude spectra, shown in Fig. 2(b). The thick black line in Fig. 2(b) shows the spectrum of the signal resulting from the averaging in the time domain. Had the alignment been unjustified in that the signals were not similar in every detail, including phase, the result would have been a low-pass filtering effect relative to the spectra of the raw, not-averaged clicks. The spectra are all very similar to the spectrum of the averaged signal in the range from 100 to ~180 kHz. Only at the frequencies outside this range, where porpoise clicks are without appreciable energy, does the spectrum of the averaged signal differ from the spectra of the clicks making it up. The in-phase averaging of the time domain signals would therefore seem to be justified.

The averaged signal has a TBP of 0.33 and an f_c of 136.9 kHz. Multiplying the envelope of the averaged signal with a sinusoid with the f_c value results in a new signal with a TBP of 0.26, which is 1.35 times lower than the original signal. The sinusoid-envelope product, with the phase that gave the best match with the averaged signal, is shown in green behind the average signal in Fig. 2(a). The correlation coefficient between these signals was 0.83. The spectrum of the artificial signal is shown together with the spectrum of the average signal in Fig. 2(b), and is clearly considerably narrower band than are the real signals.

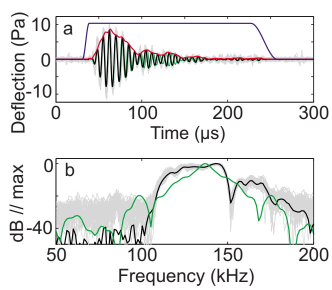

Fig. 2. (Color online) Time aligned echolocation clicks and average click. (a) The *gray lines* are the individual clicks, whereas the *black line* is the time aligned average. *Red trace* immediately above and following the signal is the envelope and the *blue line* shows the apodization function used to isolate the average click for analysis (here, the windowing function is scaled to the peak pressure value of the clicks for better visualization). In *green*, behind the average signal is shown the artificial click made by multiplying the envelope by sinusoid with the best matching phase. (b) *Gray lines* are normalized amplitude spectra of individual clicks. The *black trace* is the spectrum of the averaged click. *Green line* is the normalized spectrum of the artificial click.

The individual signals in the sequence all have higher TBPs than the artificial signal and that result is highly significant (t-test, DF=49, $P < 10^{-24}$). It is, however, not a fair test, since the superimposed noise, which was mostly gotten rid of by in-phase averaging, invariably lifts both the duration and the bandwidth to higher values. A more conservative test should be between the TBP values of the averaged signal and the artificial signal. But to make that comparison we need a measure of the variance. If, for the sake of argument, we assume that the variation in TBP of the individual clicks is due to a dominating influence on the observed variation of the underlying (unknown) noise free signals, we certainly do not underestimate that underlying variation, since the noise is bound to have some influence and that influence will result in a variation that is larger than what would have been observed, had there been no noise. Therefore, by adopting the variance figure from the population of individual TBPs and using that as the variance estimate parameter in the t-test of whether the TBP of the averaged signal is indeed smaller than the TBP obtained from the artificial signal, we produce a highly conservative test. The result of this test is that the difference is indeed also significant (t-test, DF=49, $p < 0.005$).

Several controls were made using other window lengths for the isolation of the average click, other click sequences, and single clicks. On the basis of this exploration of different analysis parameters the measured discrepancy is believed to be conservatively estimated.

The TBP of the averaged signal is four times higher than the theoretical minimum value of 0.08 held by the Gabor functions mentioned above.

5. Discussion

For harbor porpoises it seems that the hypothesis of conditional minimum TBP has therefore been falsified. The relatively high TBP value also shows that the signal is quite different from a Gabor signal.

It might well be argued that a ratio of 1.3 between model and observation is not such a bad fit, but the Wiersma study (1988) implicitly claims to explain even the absolute phase of the clicks in that the clicks portrayed have the same phase as the clicks from which the constraining values of bandwidth and delay came from. Varying the phase in the case of a porpoise click does not change the TBP, and since the claim is based mostly in these visual similarities between waveforms, that result should be cited with some caution.

It should be noted here that using waveforms from other odontocetes, similar results are found, but that in this case the argumentation above is not completely valid, since their bandwidths are above their frequency centroid, which causes the frequency centroid of the artificial signal, $\hat{s}(t)$, to be slightly different from f_c when the envelope is multiplied by $\cos(2\pi f_c t)$. It becomes dependent on the phase. Since it is difficult to imagine that a mammalian hearing system can optimally receiving a signal as broadband as the clicks from, e.g., Tursiops, it is also for that reason more relevant to consider Wiersma's (1988) hypothesis for NBHF sounds alone.

The porpoise clicks do have a low TBP, regardless of the fact that they are by no means optimal in this respect. As mentioned in the introduction, by having a low TBP one achieves that the expected signal in question is contained within a small area in a time-frequency representation. But this is only desirable if the receiver is tuned to detect signals in that same small area. Other receivers require other signals for optimal reception. Since it has been shown here that the signals do not have conditionally minimum TBP and since the receiving system is probably not tuned to receive minimum TBP signals optimally, might not the hearing system be tuned to detect the actual odontocete clicks optimally?

Again, this question is especially relevant to porpoises, where the NBHF echolocation signals might well be imagined to have evolved to be a match for the transfer function of a mammalian auditory filter. There is a single abstract (Olivieri, 2002) that reports odontocete clicks to be minimum phase (MP), and that result is confirmed here for porpoise clicks (Fig. 3). This finding is relevant in this context, because unless the signals are Gabor functions (see above) then MP signals have phase spectra that are different from the straight line that would constitute a simple delay (Biering and Pedersen, 1983). Optimal reception can be achieved only when the amplitude spectrum of the expected signal is identical to that of the transfer function of the receiving filter, and when the derivative of the phase spectrum (group delay) of the re-

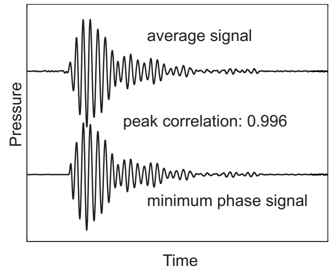

Fig. 3. Comparison between the average porpoise echolocation click and the minimum phase signal that has the same amplitude spectrum as the average click.

ceiving filter is the negation of the same quantity of the expected signal. It follows then that the matched receiving filter cannot be MP, because this would mean that the phase spectrum of both the receiver and the signal would have to be minimum, which again implies a Gabor function (Gabor, 1946; Venkatesh *et al.*, 2005), which is clearly not the case given our measured TBP value. But auditory filters are in fact also reported to be minimum phase (de Boer, 1997) and if that somewhat contended (Recio *et al.*, 1997) result holds, the echolocation clicks then cannot be matched perfectly to the impulse response of auditory filters.

It appears then that the minimum phase property in this case is merely a natural quality of a certain ubiquitous class of transient signals and that it does not represent any optimization in itself. But again, that the match between signal and receiver cannot be optimal does not mean that it is not good enough to work quite well, which the successful use of biosonar and continued existence of the animals proves.

In conclusion it may be restated that at least not all odontocetes appear to have signals that are optimized with respect to the minimal TBP under the constraining parameters of frequency centroid and signal envelope, and therefore signal delay. Consideration of MP properties of both the echolocation signals and typical auditory filters suggests that odontocete clicks in this respect do not have receivers matched to the expected signals.

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